What is the "Lee-Wick Standard Model"?

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Outline

- Foundations of Lee-Wick Electrodynamics: Unitarity & Causality
- The Lee-Wick Stadard Model: Motivations & Main Features
- Phenomenology: Signatures at Particle Colliders, Electroweak Precision tests, Flavor Physics, Neutrinos...
- Concluding Remarks

Lee-Wick Electrodynamics

T.D. Lee and G.C. Wick, Nucl. Phys. B **9**, 209 (1969)

T.D. Lee and G.C. Wick, Phys. Rev. D 2, 1033 (1970)

The photon field is replaced by the combination:

$$A_{\mu} \rightarrow A_{\mu} + iB_{\mu}$$

The Lagrange density:

$$\mathcal{L} = ar{\psi}(i\partial_\mu\gamma^\mu - m_0)\psi - rac{1}{4}(\partial_\mu A_
u - \partial_
u A_\mu)(\partial^\mu A^
u - \partial^
u A^\mu)
onumber \ -rac{1}{4}(\partial_\mu B_
u - \partial_
u B_\mu)(\partial^\mu B^
u - \partial^
u B^\mu) + rac{1}{2}\mu_0^2 B_\mu B^\mu + e_0 ar{\psi}\gamma^\mu\psi(A_\mu + iB_\mu)$$

The propagador for the coherent mixture $A_{\mu} + iB_{\mu}$:

$$\frac{1}{k^2} - \frac{1}{k^2 - \mu_0^2} = \frac{-\mu_0^2}{k^2(k^2 - \mu_0^2)} \xrightarrow[k^2 \to \infty]{} \frac{1}{k^4}$$

Finite electromagnetic mass differences between hadrons in a same isospin multiplet

Non-Hermitian Lagrangian \Rightarrow Non-Unitary S-matrix?

A different metric in Hilbert space:

$$\langle |\eta(-1)^{N_B}| \rangle > 0$$
 In a particular basis: $\eta = (-1)^{N_B}$, $A_{\mu}(B_{\mu})$ is of positive (negative) metric
 $\eta H^{\dagger} \eta = H$ Self-adjoint Hamiltonian \rightarrow real energy expectation values
 However, *H* may have complex eigenvalues
 $S^{\dagger} \eta S = \eta$ Pseudo-unitary condition for the *S*-matrix \rightarrow not satisfactory

Lee and Wick show that S is unitary if *all* stable particles are of positive metric:

 $\langle \mathbf{r} | \eta | \mathbf{r} \rangle > \mathbf{0}$ \longrightarrow for all eigenvectors of *H* with real eigenvalues

Is this condition satisfied?

The S-matrix is well defined in terms of the eigenvectors of H with real eigenvalues:

 $S_{r'r} = \langle r'^{in} | \eta | r^{out} \rangle$

The *U*-operator in the interaction picture can be decomposed:

$$U(t, -t) = U^{reg}(t, -t) + U^{exp}(t, -t)$$
 \longrightarrow diverges exponentially as t $\rightarrow \infty$

The usual relation between *U* and *S* does not hold, however:

$$\lim_{t\to\infty} U^{reg}(t,-t) = S$$

Electron-positron scattering at lowest order:



Adding corrections to the propagator:

Complex poles in unusual places \rightarrow modified distribution of contours of integration

- T.D. Lee, in Quanta, Chicago U.P., 260 (1970):
 "Naive" prescription for integration paths in Feynman integrals (not relativistically invariant)
- R. E. Cutkovsky, P.V. Landshoff, D.I. Olive and J.C. Polkinghorne, Nucl. Phys. B 12, 281 (1969): Covariant prescription, "CLOP" (not derived from a Lagrangian formulation)
- N. Nakanishi, Phys. Rev. D 3, 811 (1971): Lorentz invariance and a Lagrangian (non-perturbative) formulation seem to be not compatible
- T.D. Lee and G.C Wick, Phys. Rev. D 3, 1046 (1971): Insist in a relativistic and unitary S-matrix in spite of the lack of a Lagragian formulation
- D.G. Boulware and D.J. Gross, Nucl. Phys. B 233, 1 (1984): Functional integral for indefinite metric QFT; however, Lee-Wick theory can not be study non- pertubatively
- B.Grinstein, D.O'Connell, M.B.Wise, arXiv:0805.2156: Unitary and Lorentz invariant S-matrix in the Lee-Wick O(N) model
- A. van Tonder, arXiv:0810.1928: Lee-Wick electrodynamics is found to be unitary

Lee-Wick Electrodynamics (Causality)

S. Coleman, "Acausality", in Erice 1969, Academic Press, NY, 282 (1970)

Field operators obey the usual causality condition:

 $[\phi(x), \phi(y)] = 0$ \longrightarrow at space-like separations

These operators not only create the usual states but also states with complex energy

The right asymptotic states are obatined by subtracting the complex-energy components (*Prescriptions for contours of integration exclude outgoing exponentially growing modes*)

Violation of causality: reverse time order of physical processes

Lee-Wick Electrodynamics (Causality)

Complex-energy states show up as unusual resonances:



A word on paradoxical behaviours:

What happens if the incoming particles are stopped before they collide?

Resolution(?): Well-defined S-matrix \rightarrow natural limitations on the properties of stopping devices

The Lee-Wick Standard Model (LWSM)

B. Grinstein, D. O'Connell and M. B. Wise, Phys. Rev. D 77, 025012 (2008)

Hierarchy problem in the SM: Higgs mass corrections are quadratically sensitive to the cutoff

Tree-level Higgs mass:

$$(m_h^2)_{bare} = 2\lambda v^2$$
, $\langle H
angle = rac{v}{\sqrt{2}}$, $v \sim 247 \, {
m GeV}$

1-loop corrections from W bosons:

$$\underbrace{\frac{\sum_{h=1}^{W}}{\sum_{h=1}^{W}}}_{H} \xrightarrow{H} \underbrace{\delta m_{h}^{2}}{W} \xrightarrow{H}} \xrightarrow{W} \underbrace{\delta m_{h}^{2}}{16\pi^{2}} \Lambda^{2} \xrightarrow{H} \underbrace{If \Lambda \sim M_{\rm PL}}_{H} \xrightarrow{K} \underbrace{\delta m_{h}^{2}}{\delta m_{h}^{2}} \xrightarrow{(10^{18} \, {\rm GeV})^{2}}_{F}$$
 fine tunning

Resolution in the LWSM: one LW partner with SM couplings for each SM particle



LWSM from a Higher Order Theory

In the LWSM every field in the SM has a higher derivative term \rightarrow massive LW-particle

How does it work in a toy model?

$$\mathcal{L}_{\rm hd} = \frac{1}{2} \partial_{\mu} \hat{\phi} \partial^{\mu} \hat{\phi} - \frac{1}{2M^2} (\partial^2 \hat{\phi})^2 - \frac{1}{2} m^2 \hat{\phi}^2 - \frac{1}{4!} g \hat{\phi}^4, \qquad m >> M$$
$$\hat{D}(p) = \frac{i}{p^2 - p^4/M^2 - m^2} \longrightarrow \text{two poles} : p^2 = m, M$$

Defining an auxiliary field, $\tilde{\phi}$:

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \hat{\phi} \partial^{\mu} \hat{\phi} - \frac{1}{2} m^2 \hat{\phi}^2 - \frac{\tilde{\phi}}{\phi} \partial^2 \hat{\phi} + \frac{1}{2} M^2 \frac{\tilde{\phi}^2}{\phi} - \frac{1}{4!} g \hat{\phi}^4$$

Eliminating $ilde{\phi}$ in $\, \mathcal{L}$ using the equations of motion reproduces $\, \mathcal{L}_{
m hd} \,$

Explicit particle content through the definition $\phi = \hat{\phi} + \tilde{\phi}$:

$$\begin{split} \mathcal{L} &= \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} \partial_{\mu} \tilde{\phi} \partial^{\mu} \tilde{\phi} + \frac{1}{2} M^{2} \tilde{\phi}^{2} - \frac{1}{2} m^{2} (\phi - \tilde{\phi})^{2} - \frac{1}{4!} g (\phi - \tilde{\phi})^{4}, \\ &\longrightarrow \tilde{D}(p) = \frac{-i}{p^{2} - M^{2}} \end{split}$$

LWSM: Particle Content & Interactions (Basics)

LW-gauge bosons are massive and mix:

$$\mathcal{L}_{2g} = -\frac{1}{2} \text{Tr} \left(B_{\mu\nu} B^{\mu\nu} - \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} + W_{\mu\nu} W^{\mu\nu} - \tilde{W}_{\mu\nu} \tilde{W}^{\mu\nu} \right) - \frac{1}{2} (M_1^2 \tilde{B}_\mu \tilde{B}^\mu + M_2^2 \tilde{W}^a_\mu \tilde{W}^\mu_a) + \frac{g_2^2 v^2}{8} (W^{1,2}_\mu + \tilde{W}^{1,2}_\mu)^2 + \frac{v^2}{8} (g_1 B_\mu + g_1 \tilde{B}_\mu + g_2 W^3_\mu + g_2 \tilde{W}^3_\mu)^2$$

LW-fermions are vector-like and mix:

$$\mathcal{L}_{2\psi} = \sum_{\psi=q_L, u_R, d_R} \bar{\psi} i \, \partial \psi - \sum_{\tilde{\psi}=q, u, d} \tilde{\psi} (i \, \partial - M_{\psi}) \tilde{\psi}$$
$$-m_u (\bar{u}_R - \bar{\tilde{u}}_R) (q_L^u - \tilde{\tilde{q}}_L^u) - m_d (\bar{d}_R - \bar{\tilde{d}}_R) (q_L^d - \tilde{\tilde{q}}_L^d) + \text{h.c.}$$

Interactions between gauge bosons and fermions:

$$\begin{aligned} \mathcal{L}_{int} &= - \sum_{\psi=q_L, u_R, d_R} [g_1 \bar{\psi}(\mathcal{B} + \tilde{\mathcal{B}})\psi + g_2 \bar{\psi}(\mathcal{W} + \tilde{\mathcal{W}})\psi] \\ &+ \sum_{\psi=q, u, d} \left[g_1 \bar{\tilde{\psi}}(\mathcal{B} + \tilde{\mathcal{B}}) \tilde{\psi} + g_2 \bar{\tilde{\psi}}(\mathcal{W} + \tilde{\mathcal{W}}) \tilde{\psi} \right]. \end{aligned}$$

Experimental Signatures of LW Particles

T. G. Rizzo, JHEP **0706**, 070 (2007) T. G. Rizzo, JHEP **0801**, 042 (2008)

- Rizzo consider the resonant production of LW-gauge bosons, $W_{LW}^{\pm,0}$, B_{LW} and g_{LW} at the LHC and future e^-e^+ colliders, looking for a unique identification of the LWSM
- ◆ These processes have been analized: $pp \to (W^0_{\text{LW}}, B_{\text{LW}}) \to l^+l^- + X$, $pp \to W^{\pm}_{\text{LW}} \to l^{\pm}E_T^{miss} + X$, $pp \to g_{\text{LW}} \to jj + X$, $e^-e^+ \to (W^0_{\text{LW}}, B_{\text{LW}}) \to e^-e^+$
 - Tree-level amplitudes contain the exchange of both SM and LW-gauge bosons:

$$A\sim \frac{i}{p^2-M_{\rm SM}^2+iM_{\rm SM}\Gamma_{\rm SM}}-\frac{i}{p^2-M_{\rm LW}^2+iM_{\rm LW}\Gamma_{\rm LW}}$$

- Tree-level trilinear couplings between one LW- and two SM- gauges fileds are absent.
 LW-gluons can not be produced in gluon-gluon collisions
- The analysis of single production of LW fermions is desfavored by SM backgrounds. Pair production seem to be the dominant mechanism for LW fermions

Experimental Signatures of LW Particles (cont'd)



Transverse mass distribution for a *W*' of 1.5 TeV at the LHC Black: Lee-Wick Standard Model. Blue: Extra dimension models Red: Conventional *W*' models (SSM). Green: Left Right Sym. Model Cyan: General extra dimensional models. Yellow: SM background

Dilepton pair mass distribution at the LHC for resonances of 1.5 TeV

Green: LW Standard Model. Blue: SSM (opposite quark couplings)

Red: Sequantial Standard Model (SSM). Yellow: SM background



Experimental Signatures of LW Particles (cont'd)



Constrains from Electroweak Precision Tests

E. Álvarez, L. Da Rold, C. Schat and A.S., JHEP 0804, 026 (2008)

- We compute the oblique parameters S and T at 1-loop level
 - We integrate out the LW fields and obtain an effective Lagrangian after a field redefinition:

$$\mathcal{L}_{eff} = -\frac{1}{2} \Pi_{3B}'(0) \mathbf{W}_{\rm SM}^{\mu\nu} \mathbf{B}_{\mu\nu}^{\rm SM} + \frac{1}{2} g_2^2 \Pi_{33}(0) (\mathbf{W}_{\rm SM}^3)^2 + \frac{1}{2} g_2^2 \Pi_{11}(0) (\mathbf{W}_{\rm SM}^1)^2 + \dots$$

• Using the definitions of S and T: $S = \frac{16\pi}{g_1g_2}\Pi'_{3B}(0)$, $T = \frac{4\pi}{s^2c^2m_Z^2}[\Pi_{11}(0) - \Pi_{33}(0)]$

We obtain the tree-level contributions:

$$S = 4\pi v^2 \left(\frac{1}{M_1^2} + \frac{1}{M_2^2}\right) + \mathcal{O}\left(\frac{v^4}{M_i^4}\right), \quad T = \pi \frac{g_1^2 + g_2^2}{g_2^2} \frac{v^2}{M_1^2}$$

At 1-loop level the dominant contribution comes from quarks:



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Constrains from Electroweak Precision Tests (cont'd)

We scan the parameter space of the model and obtain bounds on the masses of LW particles



68% and 95% Confidence Level Contours in the (S, T) plane Colored dots indicate which mass M_1 , M_2 , M_q , M_u is less than 4 TeV Yellow dots from the left t right, all masses equal to 7, 6, 5, 4... TeV Black dots indicate that all M_1 , M_2 , M_q , M_u are larger than 4 TeV

Region above the curves is allowed @ 95% CL

Left: red curves (from left to right), $M_1 = M_2 = 7, 6, 5$ TeV

Right: red curves (from left to right), $M_q = M_u = 7, 6, 5, 4$



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Constrains from Electroweak Precision Tests (cont'd)

We consider an extension of the LWSM including a fourth generation of fermions and find new bounds on the LW masses



LW-gauge bosons > 5 - 8 TeV

LW-gauge bosons > 3 TeV

Other articles on electroweak constrains: T.E J. Underwood and R. Zwicky, Phys. Rev. D **79**, 035016 (2009) C.D. Carone and R.F. Lebed, Phys. Lett. B **668**, 221 (2008)

Comments on the LWSM Flavor Structure

T.R. Dulaney and M.B. Wise, Phys. Lett. B 658, 230 (2008)

The higher derivative kinetic terms contain the new flavor structure:

 $\begin{aligned} \mathcal{L}_{\mathrm{hd}}^{(\mathrm{kin})} &= \overline{\hat{Q}^{i}}_{L} i \hat{\mathcal{P}} \hat{Q}_{L}^{i} + r_{Q}^{ij} \overline{\hat{Q}^{i}}_{L} i \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{Q}}_{L}^{j} + \overline{\hat{L}^{i}}_{L} i \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{L}}_{L}^{i} + r_{L}^{ij} \overline{\hat{L}^{i}}_{L} i \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{L}}_{L}^{j} + \overline{\hat{u}^{i}}_{R} i \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{Q}}_{R}^{i} \\ + r_{U}^{ij} \overline{\hat{u}^{i}}_{R} i \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{Q}}_{R}^{i} + \overline{\hat{d}^{i}}_{R} i \hat{\mathcal{P}} \hat{\mathcal{Q}} \hat{\mathcal{Q}}_{R}^{i} + r_{D}^{ij} \overline{\hat{d}^{i}}_{R} i \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{Q}}_{R}^{j} + \overline{\hat{e}^{i}}_{R} i \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{e}_{R}^{i} + r_{E}^{ij} \overline{\hat{e}^{i}}_{R} i \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{\mathcal{P}} \hat{e}_{R}^{j} \\ \end{aligned}$

The authors integrate out the LW fermions finding flavor changing Z couplings:

$$\begin{split} \Delta \mathcal{L}_{Z} &= \sqrt{g_{1}^{2} + g_{2}^{2} Z_{\mu}} \left[\overline{u}_{R} m_{u}^{\text{diag}} \left(\mathcal{U}(u,L)^{\dagger} r_{Q} \mathcal{U}(u,L) \right) m_{u}^{\text{diag}} \gamma^{\mu} u_{R} \right. \\ &+ \overline{d}_{R} m_{d}^{\text{diag}} \left(\mathcal{U}(d,L)^{\dagger} r_{Q} \mathcal{U}(d,L) \right) m_{d}^{\text{diag}} \gamma^{\mu} d_{R} + \overline{e}_{R} m_{e}^{\text{diag}} \left(\mathcal{U}(e,L)^{\dagger} r_{L} \mathcal{U}(e,L) \right) m_{e}^{\text{diag}} \gamma^{\mu} e_{R} \\ &+ \overline{u}_{L} m_{u}^{\text{diag}} \left(\mathcal{U}(u,R)^{\dagger} r_{U} \mathcal{U}(u,R) \right) m_{u}^{\text{diag}} \gamma^{\mu} u_{L} + \overline{d}_{L} m_{d}^{\text{diag}} \left(\mathcal{U}(d,R)^{\dagger} r_{D} \mathcal{U}(d,R) \right) m_{d}^{\text{diag}} \gamma^{\mu} d_{L} \\ &+ \overline{e}_{L} m_{e}^{\text{diag}} \left(\mathcal{U}(e,R)^{\dagger} r_{E} \mathcal{U}(e,R) \right) m_{e}^{\text{diag}} \gamma^{\mu} e_{L} \Big] \end{split}$$

They also show that new flavor changing charged currents are induced:

$$\begin{split} \Delta \mathcal{L}_W &= \frac{g_2}{\sqrt{2}} W_{\mu}^{-} \left[\overline{d}_R m_d^{\text{diag}} \left(\mathcal{U}(d,L)^{\dagger} r_Q \mathcal{U}(u,L) \right) m_u^{\text{diag}} \gamma^{\mu} u_R + \frac{1}{2} \overline{d}_L V^{\dagger} m_u^{\text{diag}} \left(\mathcal{U}(u,R)^{\dagger} r_U \mathcal{U}(u,R) \right) m_u^{\text{diag}} \gamma^{\mu} u_R + \frac{1}{2} \overline{d}_L M_u^{\text{diag}} \left(\mathcal{U}(u,R)^{\dagger} r_U \mathcal{U}(u,R) \right) m_u^{\text{diag}} \gamma^{\mu} u_L + \frac{1}{2} \overline{e}_L m_e^{\text{diag}} \left(\mathcal{U}(e,R)^{\dagger} r_E \mathcal{U}(e,R) \right) m_e^{\text{diag}} \gamma^{\mu} \nu_L \right] + \text{h.c.} \end{split}$$

Comments on the LWSM Flavor Structure (cont'd)

• All the FCNC and the new FCCC couplings are suppressed by r_Q , r_L , r_U , r_D and r_E :

$$r_I = Y(ilde{I}_L) \left(rac{1}{M_I^{(ext{diag})}}
ight)^2 Y^{\dagger}(ilde{I}_L) \,, \quad I = Q, L \qquad \qquad r_J = Y(ilde{J}_R) \left(rac{1}{M_J^{(ext{diag})}}
ight)^2 Y^{\dagger}(ilde{J}_R) \,, \quad J = U, D, E$$

Neutral lepton-familly violating processes are even suppressed by the SM lepton masses:

$$\frac{\Gamma(\mu \to 3e)}{\Gamma(\mu \to e\bar{\nu}_e \nu_\mu)} \sim \left(\frac{m_e m_\mu}{1 \,\text{TeV}^2}\right)^2 \sim 10^{-21}$$

The effects can be larger if the LW and SM contributions interfere:

$$\frac{\Delta\Gamma(b\to s\nu\bar{\nu})}{\Gamma(b\to s\nu\bar{\nu})} \sim \left(\frac{m_b m_s}{1\text{TeV}^2}\right) \sim 10^{-6}$$

Corrections to the CKM matrix are small too:

$$\Delta V_{cb} \sim \frac{m_c m_t}{(1 T \, eV)^2} \sim 10^{-4}$$

Mixings from FCCC and FCNC are suppressed in non-MFV scenarios if the LW scale if ≥ 1 TeV

Neutrino masses in the LWSM

J.R. Espinosa, B. Grinstein, D. O'Connell and M.B. Wise, Phys. Rev. D 77, 085002 (2008)

Does the coupling of LW states to very heavy particles preserve the stability of the weak scale?

Seesaw mechanism explains the smallness of the observed neutrino masses:

$$m_
u \sim rac{v^2}{m_R} \qquad m_R \uparrow \quad m_
u \downarrow \qquad m_R \gtrsim 10^{11}\,{
m TeV}$$

1-loop right handed neutrino correction to the Higgs mass:



If $m_R \gtrsim 10^4 \,\mathrm{TeV}$ corrections are too large

If $g_Y M_L \lesssim 10$ TeV corrections are small enough

Other works on the LWSM

- F. Wu and M. Zhong, Phys. Lett. B 659, 694 (2008); F. Wu and M. Zhong, Phys. Rev. D 78, 085010 (2008): LW higher derivative terms are induced by gravitational radiative corrections
- ◆ F. Krauss, T.E.J. Underwood and R. Zwicky, Phys. Rev. D **77**, 015012 (2008): Distinctive LW experimental signatures through the process $gg \rightarrow H \rightarrow \gamma\gamma$
- B. Grinstein, D. O'Connell and M. B. Wise, Phys. Rev. D 77, 065010 (2008): Scattering of massive LW-vector bosons does not violate perturbative unitarity
- B. Grinstein and D. O'Connell, Phys. Rev. D 78, 105005 (2008):
 1-loop renormalization of LW gauge theory
- C.D. Carone and R.F. Lebed, JHEP 0901, 043 (2009): Next-to-minimal higher-derivative LWSM
- S. Lee, arXiv:0810.1145:
 LW fields as a candidate of dark energy
- Y.F. Cai, T.t. Qiu, R. Brandenberger and X.m. Zhang, arXiv:0810.4677: LWSM as possible solution of the cosmological singularity problem

and more...

Conclusions

- The Lee-Wick Standard Model cures the hierarchy problem by adding one degree of freedom for each SM field
- The LW partners have the same spin-statistics as the SM fields but quadratic terms with opposite signs
- The presence of negative-norm states anticipated conflicts with unitarity. Contours of integration have to be redefined. It appears that this can be done perturbatively in a way that preserves unitarity
- The theory is not causal at short time scales
- Unique experimental signals seem to be clearer at electron-positron colliders
 than at the LHC
- Electroweak precision tests contrain LW particles to be $\gtrsim 2.5 \text{ TeV}$
- FCNC and FCCC are induced in the theory but their effecs are small
- Right-handed neutrinos with Majorana masses coupled to the LW fields do not destabilize the electroweak scale