Late Time Cosmic Acceleration

Md. Wali Hossain

Centre for Theoretical Physics

JMI, New Delhi, India

Department of Physics, Universitè de Montrèal, Montrèal, Canada

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Plan

- Standard Cosmology
- Observational Evidences
- Theoretical Models

Cosmic History

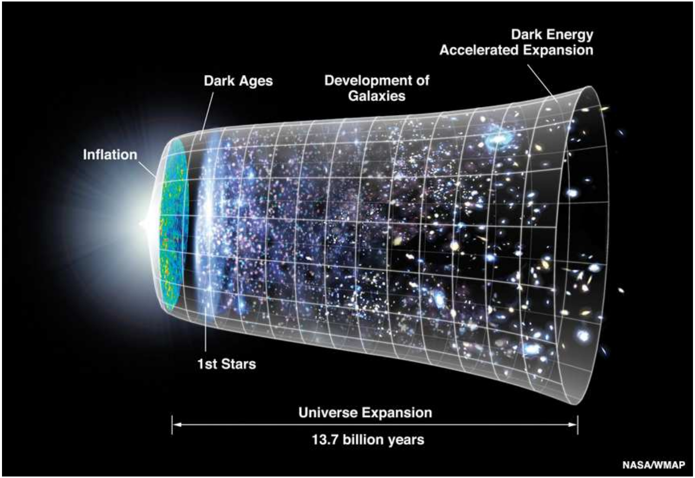

Figure : Cosmic history. Picture is taken from wfirst.gsfc.nasa.gov.

Cosmological Scale $>$ 100 Mpc. $\quad$ $1\text{Mpc} = 3 \times 10^{22}$ m.

Standard Cosmology

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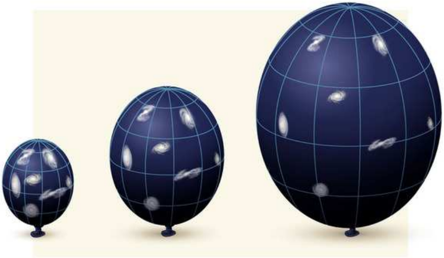
Cosmological Principles: Viewed on a sufficiently large scale, Universe looks same in all directions for all observers.

- No preferred directions $\Longrightarrow$ Isotropy.
- One part of the Universe is approximately same with any other part $\Longrightarrow$ Homogeneity.

Universe is expanding $\Longrightarrow$ One of the most important discoveries in cosmology.



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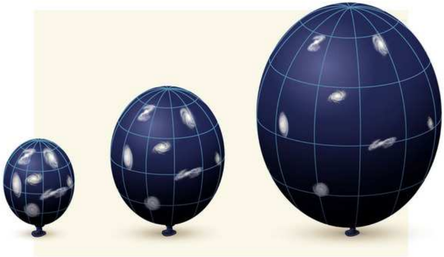
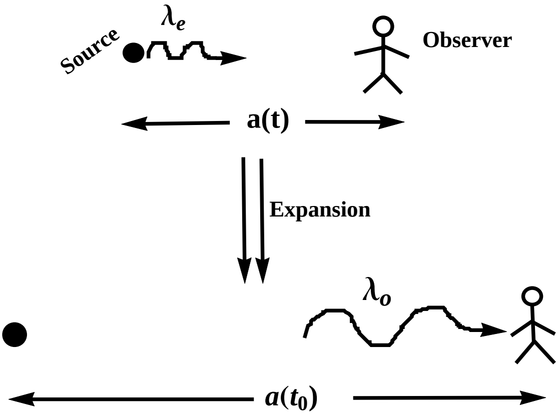


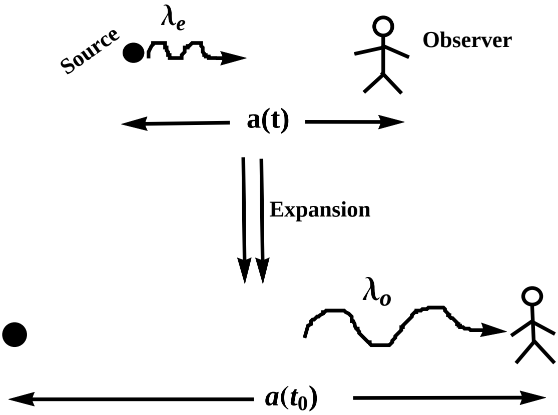
Hubble's Law:

$$v_{\mathrm{r}} = H_0 D \, .$$

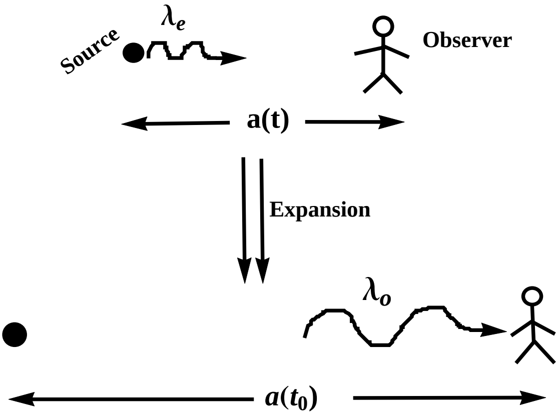
v_{r} = recessional velocity,
D = the proper distance (which can change over time, unlike the comoving distance which is constant) from the galaxy to the observer,
H_0 = a constant known as Hubble's constant.



Source
λ_e
Observer
a(t)
Expansion
λ_o
$a(t_0)$



$$\frac{\lambda_{\mathrm{o}}}{\lambda_{\mathrm{e}}} = \frac{a(t_0)}{a(t)}$$



$$\frac{\lambda_o}{\lambda_e} = \frac{a(t_0)}{a(t)} \implies \text{Redshift, } z = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{a(t_0)}{a(t)} - 1.$$

$a(t) \implies$ Scale factor.

Standard Cosmology

For small redshift $\Longrightarrow$ $\boxed{cz \approx H_0 D = v_{\mathrm{r}}}$.

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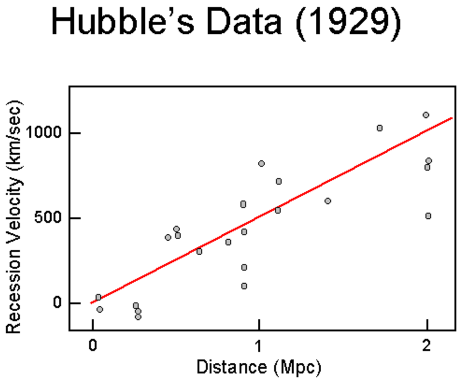

Figure : In 1929. Taken from www.astronomy.ohio-state.edu/ pogge/.

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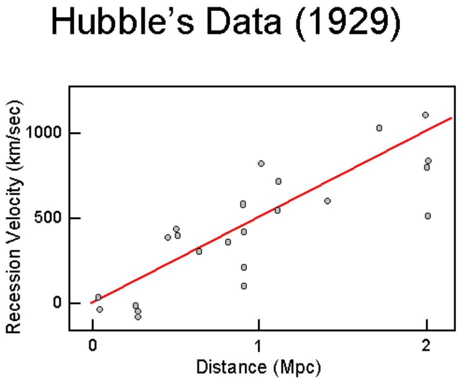

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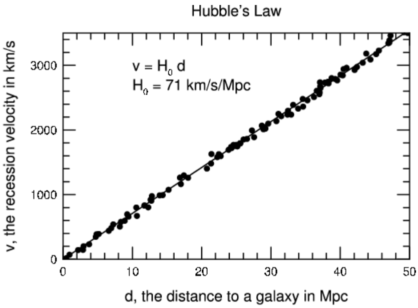


Figure : Recent result. Taken fom firedrake.bu.edu.

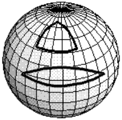
Friedmann–Lemaître–Robertson–Walker (FLRW) metric:

Expanding homogeneous and isotropic Universe can be represented by,

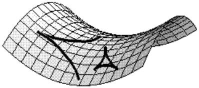
$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right),$$

$k \Longrightarrow$ A constant representing the curvature of the space $\Longrightarrow$

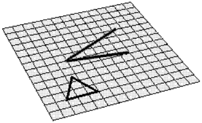
- $k > 0 \Longrightarrow$ Closed Universe.
- $k < 0 \Longrightarrow$ Open Universe.
- $k = 0 \Longrightarrow$ Flat Universe.



Universe with *positive* curvature. Diverging line converge at great distances. Triangle angles add to more than 180°.



Universe with *negative* curvature. Lines diverge at ever increasing angles. Triangle angles add to less than 180°.



Universe with no curvature. Lines diverge at constant angle. Triangle angles add to 180°.

Taken from www.astronomynotes.com.

Solving Einstein's equation of GR $\Longrightarrow$ Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\rho - \frac{k}{a^2}\,.$$

$H =$ Hubble parameter $= \dot{a}/a$ and we have taken $c = 1$.

$M_{\mathrm{Pl}}^2 = 1/8\pi G =$ Planck mass.

$\rho = \rho_{\mathrm{m}} + \rho_{\mathrm{r}} + \rho_{\Lambda} =$ total density.

We can also define curvature density $\Longrightarrow$ $\rho_{\mathrm{k}} = 3M_{\mathrm{Pl}}^2 k/a^2$. Observations suggest $\rho_{\mathrm{k}} \to 0$ $\Longrightarrow$ Makes life simpler.

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Raychaudhuri equation,

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{6M_{\rm Pl}^2}\left(\rho + 3p\right) \, .$$

$p =$ pressure.
Pressure due to curvature term $\Longrightarrow$ $p_{\rm k} = kM_{\rm Pl}^2/a^2$ $\Longrightarrow$ $\rho_{\rm k} + 3p_{\rm k} = 0$.

Some important cosmological parameters:

- Hubble parameter $\Longrightarrow$ $H = \dot{a}/a$. Present value from *Planck* + WP $\Longrightarrow$ $H_0 = 67.3 \pm 1.2\ \mathrm{km\ s^{-1}Mpc^{-1}}$.

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- ρ_c $\Longrightarrow$ Critical density $= 3H^2 M_\mathrm{Pl}^2$.
  Present value $\Longrightarrow$ $\rho_{\mathrm{c}0} \sim 10^{-47}\ \mathrm{GeV}^4$.
- Density parameter $\Longrightarrow$ $\Omega = \rho/\rho_\mathrm{c}$.
- Equation of state $\Longrightarrow$ $w = \dfrac{\mathit{Pressure}}{\mathit{Density}}$.
  For matter $\Longrightarrow$ $w_\mathrm{m} = 0$,
  For radiation $\Longrightarrow$ $w_\mathrm{r} = 1/3$.
  For cosmological constant $\Longrightarrow$ $w_\Lambda = -1$.

constraint on the curvature term:

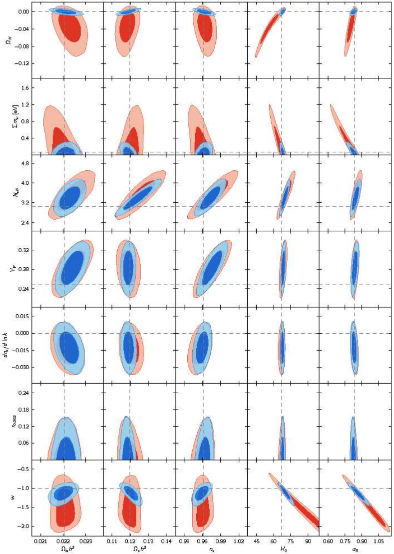


Figure : P. A. R. Ade et al., A&A 571, A16 (2014). Red for *Planck* + *WP* and blue for *Planck* + *WP* + *BAO*.

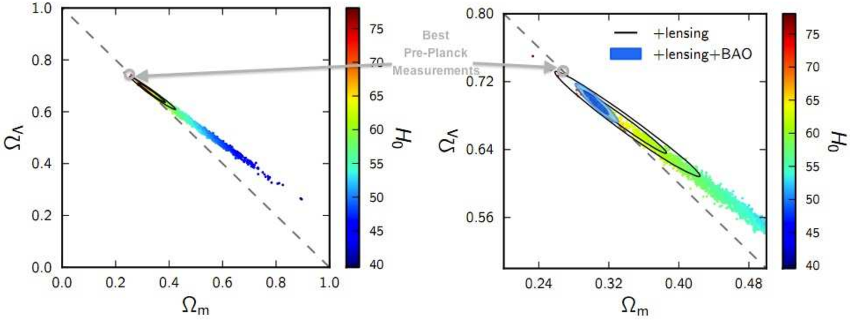
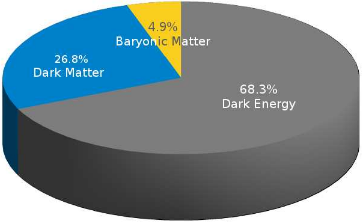


Figure : colored contours are for *Planck* + *WP* + *highL* (colour-coded by the value of H0). Black line contours are for *Planck* + *WP* + *highL* + *lensing*. Blue contours are for *Planck* + *WP* + *highL* + *lensing* + *BAO*. *Planck* Collaboration: P. A. R. Ade et al., A&A 571, A16 (2014). Edited picture is taken from scienceblogs.com/startswithabang/2013/05/23/what-is-dark-energy-2/.

Components of the Universe:

- Matter $\Longrightarrow$ $\Omega_{\rm m0} = 0.31$.
- Radiation $\Longrightarrow$ $\Omega_{\rm r0} \sim 10^{-4}$.
- Dark energy $\Longrightarrow$ $\Omega_{\rm DE0} = 0.69$.
- Spatial curvature $\Longrightarrow$ $\Omega_{\rm k0}$ is Nearly zero.



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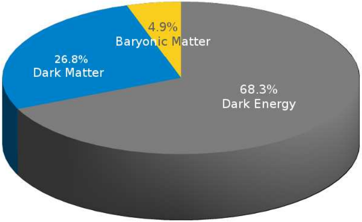

Total density $\Longrightarrow$ $\Omega = \Omega_{\mathrm{m}} + \Omega_{\mathrm{r}} + \Omega_{\mathrm{DE}} + \Omega_{\mathrm{k}}$
$\Omega = 1$ when $\Omega_{\mathrm{k}} = 0$ $\Longrightarrow$ Flat Universe.

Age crisis and cosmological constant:

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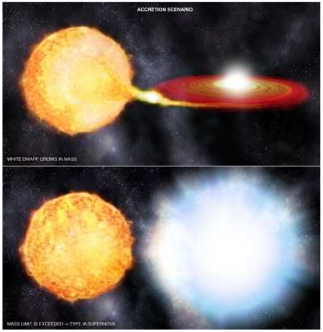
Age of the Universe $\Longrightarrow$ 13.8 Gyrs (Planck+WP+highL+BAO).

Observational Evidences of Dark Energy

Type Ia Supernovae $\Longrightarrow$ occur in binary systems in which one of the stars is a white dwarf while the other can vary from a giant star to an even smaller white dwarf.



White dwarf accretes matter from a companion $\Longrightarrow$ Exceeds the Chandrasekhar limit ($1.44 M_\odot$) $\Longrightarrow$ The electron degeneracy pressure fails to support the gravitational pressure $\Longrightarrow$ Temperature increases due to compression $\Longrightarrow$ Carbon fusion $\Longrightarrow$ Type Ia supernova.

- Their intrinsic luminosity is know $\Longrightarrow$ Standard candles.
- Their apparent luminosity can be measured.

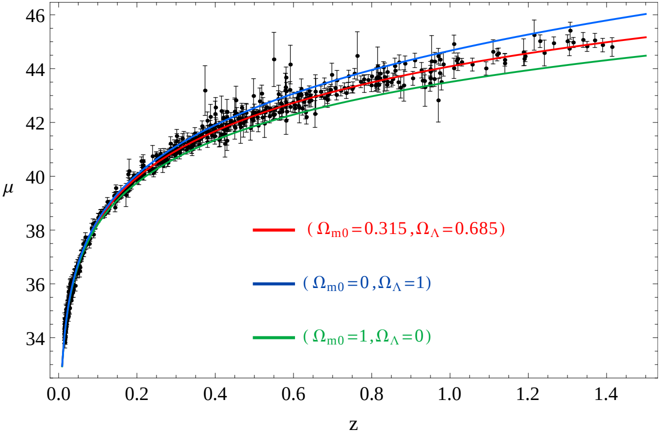
Using these two, observations gives us the distance modulus,

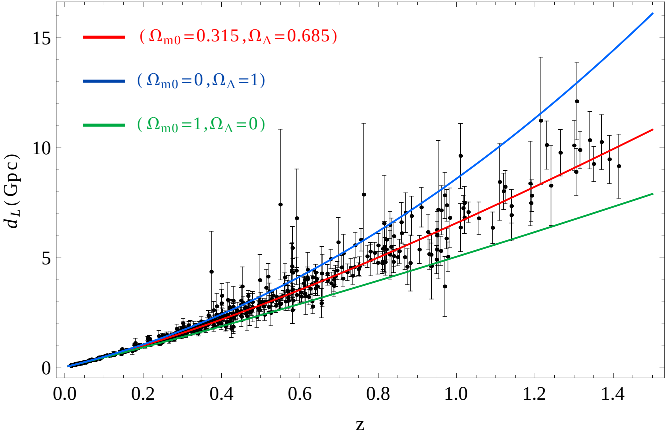
$$\mu = 5\log_{10}\left(\frac{d_{\mathrm{L}}}{\mathrm{Mpc}}\right) + 25\,.$$

Where,

$$d_{\mathrm{L}} = (1+z)\int_0^z \frac{\mathrm{d}z'}{H(z')}\,.$$

is called the luminosity distance.


46
44
42
40
38
36
34
μ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
z
$(\Omega_{m0}=0.315, \Omega_{\Lambda}=0.685)$
$(\Omega_{m0}=0, \Omega_{\Lambda}=1)$
$(\Omega_{m0}=1, \Omega_{\Lambda}=0)$



$(\Omega_{m0}=0.315,\Omega_\Lambda=0.685)$
$(\Omega_{m0}=0,\Omega_\Lambda=1)$
$(\Omega_{m0}=1,\Omega_\Lambda=0)$
d_L (Gpc)
z
0
5
10
15
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

What Is Dark Energy?

Which causes the acceleration of the Universe $\Longrightarrow$ Gives $-$ve pressure $\Longrightarrow \ddot{a} > 0$.

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So $\ddot{a} > 0 \Longrightarrow p/\rho < -1/3 \Longrightarrow w < -1/3$.

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So $\ddot{a} > 0 \Longrightarrow p/\rho < -1/3 \Longrightarrow w < -1/3$.

After *Planck* 2013 results constraint on the equation of stat of dark energy is $w = -1.13^{+0.24}_{-0.25}$ (2σ limits).

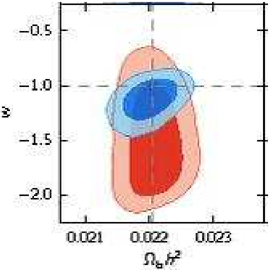

Figure : Planck+WP (red) and Planck+WP+BAO (blue)

Present Universe is dark energy dominated $\Longrightarrow$ $\Omega_{\rm m0} \approx 0.3$ and $\Omega_{\rm DE} \approx 0.7$.

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Dark energy $\Longrightarrow$ Cosmological constant Λ
$\Longrightarrow$ Equation of state is -1 $\Longrightarrow$ within the *Planck* bound.
$\Longrightarrow$ Solves the problem of age crisis of the Universe
$\Longrightarrow$ Fits with data very well.

Cosmological Constant As The Dark Energy

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$\Longrightarrow$ Fine tuning problem

$$\Longrightarrow \boxed{\frac{\rho_{\Lambda,\rm obs}}{\rho_{\Lambda,\rm theo}} = 10^{-120}} \quad \text{and} \quad \boxed{\rho_\Lambda \approx \rho_{\rm m0}}\,.$$

cosmic Coincidence

Scalar Field as Dark Energy

One can also think of dynamical dark energy where the energy density of the dark energy component varies with time $\Longrightarrow$ *e.g.*, a slowly rolling scalar field $\Longrightarrow$ $\boxed{\text{QUINTESSENCE}}$:

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$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\,.$$

In flat FRW background:

Density $\Longrightarrow$ $\rho_\phi = (1/2)\dot{\phi}^2 + V$

Pressure $\Longrightarrow$ $p_\phi = (1/2)\dot{\phi}^2 - V$.

- Equation of state of the scalar field is given by,

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V}\,.$$

And the energy density,

$$\rho_\phi = \rho_{\phi 0} \mathrm{e}^{-3\int(1+\mathrm{w}_\phi)\mathrm{da}/\mathrm{a}}\,.$$

$\Longrightarrow$ Negative pressure can be achieved when $\dot{\phi}^2 < 2V$.

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$\Longrightarrow$ Negative pressure can be achieved when $\dot{\phi}^2 < 2V$.

- If $\dot{\phi}^2 \gg V(\phi) \Longrightarrow w_\phi \approx 1 \Longrightarrow \rho_\phi \sim a^{-6}$.

If $V(\phi) \gg \dot{\phi}^2 \Longrightarrow w_\phi \approx -1 \Longrightarrow \rho_\phi \approx \mathrm{Constant}$.

So $\rho_\phi \sim a^{-n}$ where $0 \leq n \leq 6$.

- Two kind of behavior $\Longrightarrow$ Tracker and Thawing.

Tracker

Scalar field tracks the background during the radiation and matter era and take over matter at recent past $\Longrightarrow$ Late time solution is an attractor for a wide range of initial conditions

P. J. Steinhardt, L. -M. Wang and I. Zlatev, PRD **59**, 123504 (1999)

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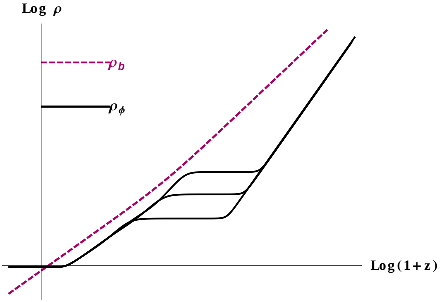


Figure : Schematic diagram of tracker behavior

- All paths are converging to a common evolutionary track.
- Not all potential can give rise to tracker behavior $\Longrightarrow$ A limitation.
- $\Gamma > 1$ where $\Gamma = \dfrac{V''V}{V'^2}$.
- Runaway potentials like $\dfrac{1}{\phi^n}$ or exponential potential $\mathrm{e}^{\mathrm{M}/\phi}$ can give rise to tracker solution.
- Field's EoS goes towards -1.

Tracker

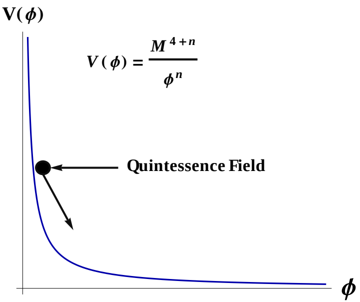


Figure : Schematic diagram of inverse power law potential, a runaway potential.

- Steep nature of the potential is needed. Hubble friction $3H\dot{\phi}$ increases since $\dot{\phi}$ increases while rolling down the steep region of the potential $\Longrightarrow$ Field's evolution freezes and energy density becomes comparable with the background energy density $\Longrightarrow$ Field starts evolving and follow the background up to recent past.
- Along the common evolution path field's EoS nearly follows the background EoS and $w_\phi \approx \dfrac{w_{\rm B} - 2\,(\Gamma - 1)}{(2\Gamma - 1)}$. For $V \sim 1/\phi^n$ during matter era $w_\phi = -\dfrac{2}{n+2}$.
- Some potentials which reduce to inverse power law and exponential nature asymptotically can also give tracker solution $\Longrightarrow$ Example: Double exponential or cos hyperbolic

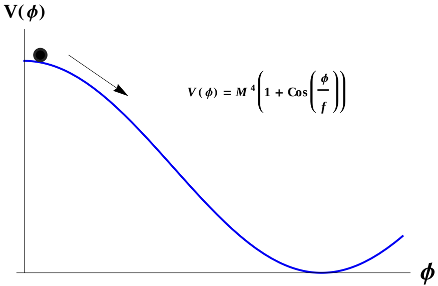


Figure : Schematic diagram of the potential leads to thawing behavior.

- Field's energy density remains constant during the early time for huge Hubble damping.
- Field's EoS starts moving away from -1 towards higher values from the recent past.
- Dark energy can be transient as the field starts evolving at the recent past.
- There is no common path of evolution and the system is very much sensible to the initial conditions.

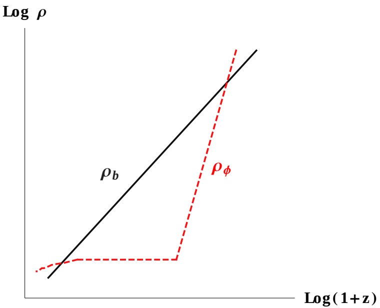
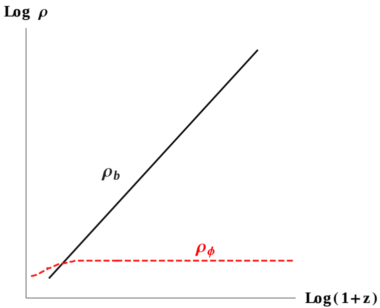


Figure : Schematic diagram of thawing behavior.



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Extended Quintessence:

$$\mathcal{S} = \mathcal{S}_{\rm EH} + \mathcal{S}_{\rm m}\left(\mathcal{C}(\phi) g_{\alpha\beta}; \Psi_{\rm m}\right) .$$

$\mathcal{C}(\phi) = {\rm e}^{2\beta\phi/{\rm M_{Pl}}}$, where β is the coupling constant$=0.036 \pm 0.016$ (*Planck*,WP,BAO).

V. Pettorino, PRD **88**, no. 6, 063519 (2013)

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$F(R)$ gravity:

Ricci scalar in Einstein Hilbert term is replaced by a function of R
$\Longrightarrow$ Can be transformed to coupled quintessence by doing a conformal transformation from Jordan frame to Einstein frame.

A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010)

Galileon:

A. Nicolis, R. Rattazzi and E. Trincherini, PRD **79**, 064036 (2009)

$$\mathcal{L}^{(1)} = \phi$$
$$\mathcal{L}^{(2)} = (\partial_\mu\phi)^2$$
$$\mathcal{L}^{(3)} = (\partial_\mu\phi)^2\Box\phi$$
$$\mathcal{L}^{(4)} = (\partial_\mu\phi)^2[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2]$$
$$\mathcal{L}^{(5)} = (\partial_\mu\phi)^2[(\Box\phi)^3 - 3(\partial_\mu\partial_\nu\phi)^2\Box\phi + 2\partial_\mu\partial_\nu\phi\partial^\nu\partial^\alpha\phi\partial_\alpha\partial^\mu\phi]$$

- Galileon Has higher derivative terms in the Lagrangian.
- Possesses shift symmetry ($\phi \to \phi + b_\mu x^\mu + c$) in Minkowski background.
- Gives second order EoM.
- Can preserve local physics through Vainshtein mechanism.
- Has superluminality problem.

Massive Gravity:

C. de Rham, G. Gabadadze and A. J. Tolley, PRL **106**, 231101 (2011)

$$\mathcal{S}_{\rm mg} = \frac{m^2 M_{Pl}^2}{8} \int \mathrm{d}^4 x \sqrt{-g} \left[U_2 + \alpha_3 U_3 + \alpha_4 U_4 \right],$$

where,

$$\begin{aligned} U_2 &= 4([\mathcal{K}]^2 - [\mathcal{K}^2]) \\ U_3 &= [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \\ U_4 &= [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4], \end{aligned}$$

and

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\partial_\alpha \phi^a \partial_\nu \phi^b \eta_{ab}},$$

Models for Late Time Acceleration

PROBLEM $\Longrightarrow$ Does not give cosmology $\Longrightarrow$ Scale factor a appears to be a constant. SOLUTION $\Longrightarrow$ Extended nonlinear massive gravity $\Longrightarrow$ Quasidilaton theory and Mass Varying Massive Gravity. In Quasidilaton theory,

G. D'Amico, G. Gabadadze, L. Hui and D. Pirtskhalava, PRD **87**, 064037 (2013)
R. Gannouji, MWH, M. Sami and E. N. Saridakis, PRD 87, **123536** (2013)

$$\mathcal{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - e^{\sigma/M_{Pl}} \sqrt{g^{\mu\alpha}\partial_{\alpha}\phi^{a}\partial_{\nu}\phi^{b}\eta_{ab}}\,,$$

In Mass Varying Massive Gravity:

Q. -G. Huang, Y. -S. Piao and S. -Y. Zhou, PRD **86**, 124014 (2012)

Graviton mass term m^2 is replaced by a scalar field potential $V(\phi)$. Another potential is also added to get the late time acceleration.

Scalar field as Inflaton:

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Inflation $\Longrightarrow$ Early accelerated phase of the Universe.

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Where $V(\phi)$ is the potential.

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Friedmann equation in flat FRW background,

$$3H^2M_{\rm Pl}^2 = \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$(2\dot{H} + 3H^2)M_{\rm Pl}^2 = -p_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Scalar field equation of motion,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0$$

Slow Roll:

During slow roll: $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$

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$$\implies H^2 \approx V/3M_{\rm Pl}^2 \text{ and } \dot{\phi} = -V'/3H.$$

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Slow roll parameters:

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}\phi}\right)^2 \qquad \eta = \frac{M_{\rm Pl}^2}{V}\frac{\mathrm{d}^2 V}{\mathrm{d}\phi^2}$$

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Slow roll condition:

$$\boxed{\epsilon, \eta \ll 1}$$

Inflation

$\epsilon \approx \eta \approx 1 \implies$ End of inflation $\implies$ Reheating $\implies$ Potential needs a minimum.

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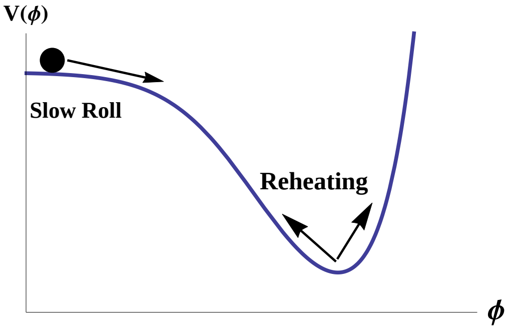

Figure : Schematic diagram of inflaton potential

Expected nature of the potential

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- One for inflation and
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$\Longrightarrow$ In between two flat regions we need steep region of the potential to have tracker behavior.

Quintessential Inflation

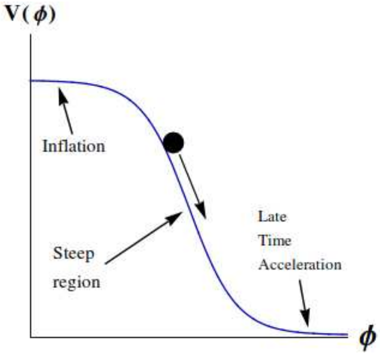

Figure : Schematic diagram of an effective potential which can give quintessential inflation.

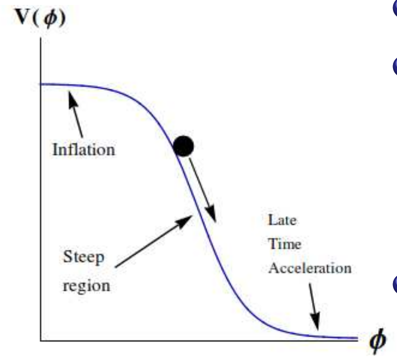

Figure : Schematic diagram of an effective potential which can give quintessential inflation.

Problems

1. Find out a suitable potential.
2. Scalar field survives until late times $\Longrightarrow$ potential is typically of a run-away type $\Longrightarrow$ One requires an alternative mechanism of reheating *e.g.*, instant preheating.
3. Long kinetic regime enhances the amplitude of relic gravitational waves $\Longrightarrow$ violates nucleosynthesis constraints at the commencement of radiative regime.

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CASE I Models in which the field potential has a required steep behavior for most of the history of universe but turn shallow at late times, for instance, the inverse power-law potentials $\Longrightarrow$ Use brane damping term for inflation $\Longrightarrow$ Brane inflation.

How to build the unified picture?

CASE I Models in which the field potential has a required steep behavior for most of the history of universe but turn shallow at late times, for instance, the inverse power-law potentials $\Longrightarrow$ Use brane damping term for inflation $\Longrightarrow$ Brane inflation.

CASE II Models in which the field potential is shallow at early epochs giving rise to inflation, followed by the required steep behavior
$\Longrightarrow$ Coupling between massive neutrinos and scalar field
$\Longrightarrow$ Variable gravity.

CASE I

- Invoke Randall-Sundrum (RS) braneworld corrections to facilitate inflation with steep potential at early epochs.

- As the field rolls down to low energy regime, the braneworld corrections disappear, giving rise to a graceful exit from inflation and thereafter the scalar field has the required behavior.

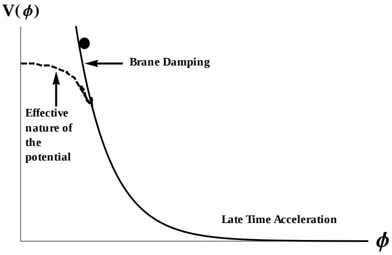

Figure : Schematic diagram of an effective potential of quintessential inflation with brane damping term.

CASE II

Potential is flat at the early epoch $\Longrightarrow$ Gives inflation.

During lat time $\Longrightarrow$ Coupling between massive neutrinos and scalar field forms an effective potential which has a minimum $\Longrightarrow$ Field oscillates around the minimum and eventually settles down $\Longrightarrow$ Scalar of dark energy is set by the massive neutrino mass scale.

C. Wetterich, PRD **89**, 024005 (2014),
MWH, R. Myrzakulov, M. Sami and E. N. Saridakis, PRD **90**, 023512 (2014)

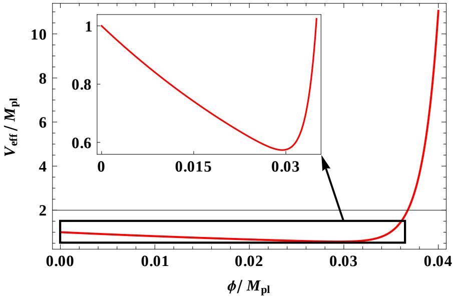

Figure : Effective potential.

Summary

- Observations confirm that recent era is dark energy dominated.
- Cosmological constant is the best model model till though it has some theoretical issues.
- There lot of models on late time acceleration but no model can solve the problem associated with cosmological constant.

THANK YOU